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Combining physics models and Gaussian processes for traffic prediction

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Joint work with Mickaël Binois[1] and Paola Goatin[1]

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Motivation

Source: https://www.larochellegc.com/realisations/analyse-congestion-routiere-avantagee-cloud/

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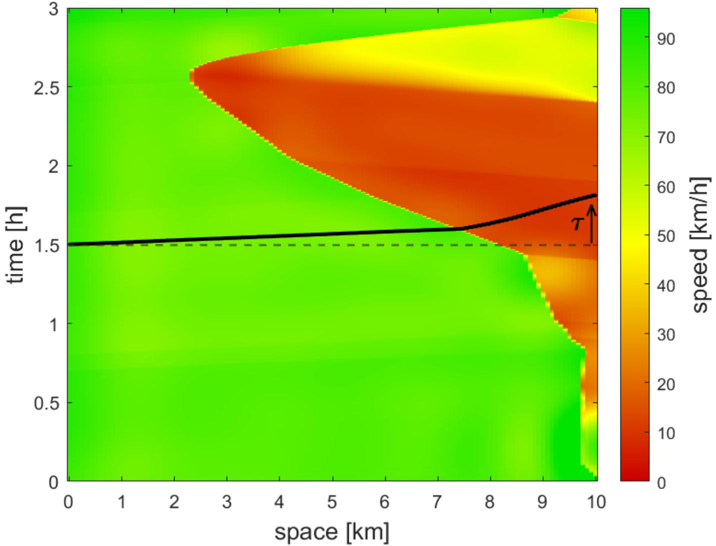

Illustration of vehicle trajectory and travel time τ for sample scenario.

Outline

1. Macroscopic traffic flow models
2. Calibration approaches
3. Synthetic traffic data
4. Traffic estimation and prediction

Outline

Macroscopic traffic flow models

Consideration of **averaged and aggregated** traffic quantities such as:

- traffic **density** ρ (number of vehicles per space unit),
- vehicle **velocity** v (distance covered by vehicles per time unit),
- traffic **flow** $q = \rho v$ (number of vehicles per time unit).

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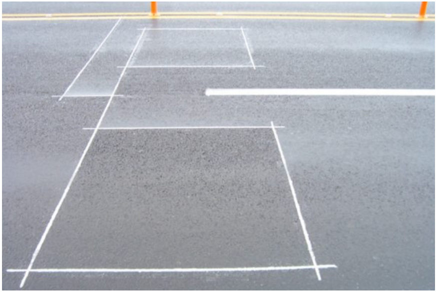
Data can be **measured** by:

- magnetic loop detectors,
- video recordings,
- wireless sensor networks,
- etc.



Source: https://www.ecm-france.com/en/areas-of-activity/weigh-in-motion/inductive-loop/

Generic Second Order Model (GSOM)

We consider a **Generic Second Order traffic flow Model (GSOM)**[2] consisting in a 2×2 hyperbolic system of conservation laws

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0, \\ \partial_t(\rho w) + \partial_x(\rho w v) = 0, \end{cases} \qquad x \in \mathbb{R},\ t > 0, \tag{1}$$

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where

$\rho = \rho(t, x)$ represents the density,

$w = w(t, x)$ the Lagrangian vehicle property,

$v = \mathcal{V}(\rho, w)$ the average speed of vehicles.

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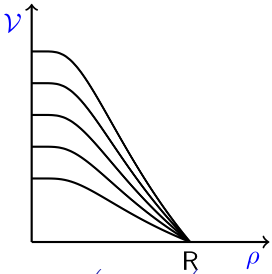

$\mathcal{V}(\rho, w) = w \left(1 - exp\left(\frac{C}{V}(1 - \frac{R}{\rho})\right)\right)$

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Initial Boundary Value Problem (IBVP)

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$$(\rho, w)(0, x) = (\rho_0, w_0)(x), \qquad x \in]x_{in}, x_{out}[,$$

$$(\rho, w)(t, x_{in}) = (\rho_{in}, w_{in})(t), \qquad t > 0,$$

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$$(\rho, w)(t, x_{out}) = (\rho_{out}, w_{out})(t), \qquad t > 0,$$

with values in an invariant domain of the form

$$\Omega := \left\{ U = (\rho, w) \in \mathbb{R}^2 \colon \rho \in [0, R(w_{max})], w \in [w_{min}, w_{max}] \right\}$$

for some $0 < w_{min} \leq w_{max} < +\infty$.

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We denote by F the so-called "**field**" where P, the **real process** under study, is physically observed.

It is generally assumed that P and F are related by:

$$y^F(t,x) = y^P(t,x) + \varepsilon \text{ where } \varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2).$$

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1. L^2 - approach:

Mathematical model, M, represents perfectly the real system:

$$y^F(t,x) = y^M(t,x,\theta^*) + \varepsilon$$

with θ^* being the **optimal calibration parameter** computed by:

$$\theta^* = \operatorname*{argmin}_{\theta} \sum_{(t,x)\in(T,X)} \left| y^F(t,x) - y^M(t,x,\theta) \right|^2.$$

2. Kennedy-O'Hagan[4] (KOH) - approach:

Model the **inadequacy** between the **mathematical model** and the **reality** by a **discrepancy** (bias) term:

$$\underbrace{y^F(t,x)}_{\text{observation}} = \underbrace{y^M(t,x,\theta^*)}_{\text{simulation}} + \underbrace{\tilde{b}(t,x,\theta^*)}_{\text{discrepancy}} + \underbrace{\varepsilon}_{\text{obs. error}}$$

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where

$\mathrm{b_n} = \tilde{b} + \varepsilon \sim \mathcal{N}\big(0, \mathrm{K_n}(\lambda)\big)$ with covariance $\mathrm{K_n}(\lambda) = \sigma^2\big(\mathrm{C_n}(l_1, l_2) + g\mathrm{I_n}\big)$,
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θ^* the **optimal calibration parameter** computed by:

$$\theta^* = \underset{\theta}{\operatorname{argmax}} \frac{1}{\sqrt{(2\pi)^n |\mathrm{K_n}(\lambda)|}} \exp\left(-\frac{1}{2}\mathrm{b_n}(\theta)^\top \big(\mathrm{K_n}(\lambda)\big)^{-1}\mathrm{b_n}(\theta)\right).$$

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Gaussian process modeling

Given: n bias-observations $\mathrm{b_n} = b(\mathcal{X}_n)$ at $\mathcal{X}_n = \big((t_1, x_1), \dots, (t_n, x_n)\big)$
$\rightarrow$ at $\hat{n}$ new locations $\hat{\mathcal{X}}_{\hat{n}} = \big((\hat{t}_1, \hat{x}_1), \dots, (\hat{t}_{\hat{n}}, \hat{x}_{\hat{n}})\big)$ it holds:

$$\mathrm{b}(\hat{\mathcal{X}}_{\hat{n}}) \mid \mathrm{b_n} \sim \mathcal{N}(m_n(\hat{\mathcal{X}}_{\hat{n}}), s_n^2(\hat{\mathcal{X}}_{\hat{n}}, \hat{\mathcal{X}}_{\hat{n}})),$$

$$m_n(\hat{\mathcal{X}}_{\hat{n}}) := \mathbb{E}[\mathrm{b}(\hat{\mathcal{X}}_{\hat{\mathrm{n}}})|\mathrm{b_n}] = \mathrm{k_n}(\hat{\mathcal{X}}_{\hat{n}})^\top \mathrm{K_n}^{-1}\mathrm{b_n},$$

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where $k(\cdot,\cdot) = \sigma^2 \mathbb{C}orr\big(\mathrm{b}(\cdot), \mathrm{b}(\cdot)\big)$, $\mathrm{k}_n(\hat{\mathcal{X}}_{\hat{n}}) := \big(k(\hat{\mathcal{X}}_{\hat{n}}^{(j)}, \mathcal{X}_n^{(i)})\big)_{1\le j\le \hat{n}, 1\le i\le n}$.

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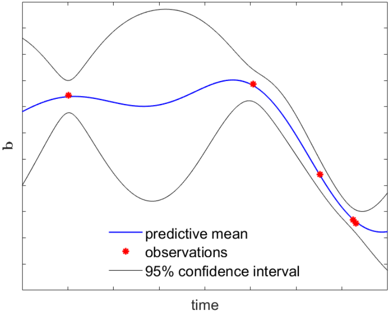

Gaussian kernel correlation matrix

Standard formula:

$$\mathbb{C}orr\big(\mathrm{b}(t,x),\mathrm{b}(t',x')\big) = \exp\left(-\frac{(t-t')^2}{l_1^2}\right)\exp\left(-\frac{(x-x')^2}{l_2^2}\right)$$

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→ Slight improvement observed

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→ No improvement observed (only approximation for derivative available)

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Generation of traffic data by a **microscopic simulator** SUMO[6]:

[6] Lopez et al. *Microscopic Traffic Simulation using SUMO*, 2018

Generation of traffic data by a **microscopic simulator** SUMO[6]:

- 10 km long road stretch,
- 3 lanes, no ramps,
- speed limit: 100 km/h,
- different classes of vehicle types (normal, fast and slow cars).

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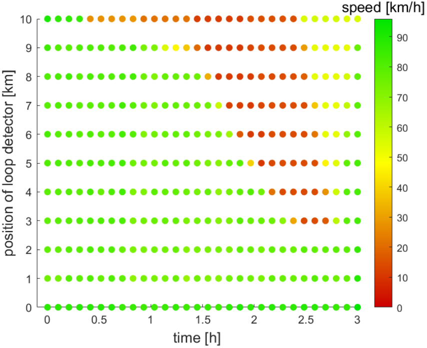

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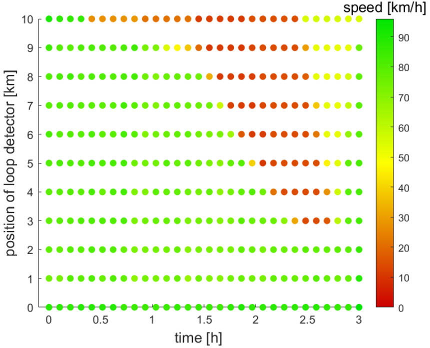

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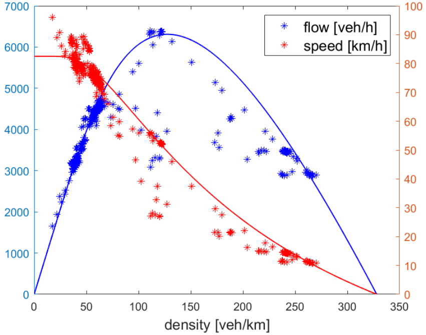

with $\theta^*_{L^2} = (83, 48, 328)$.

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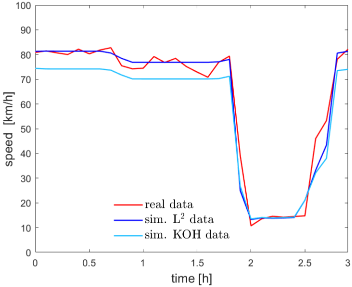

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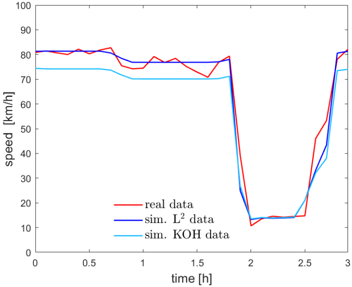

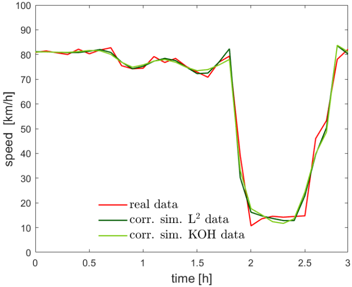

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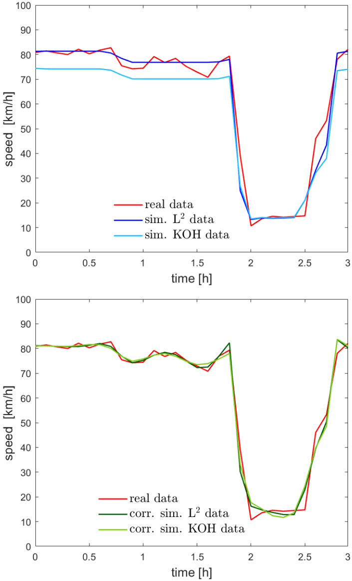

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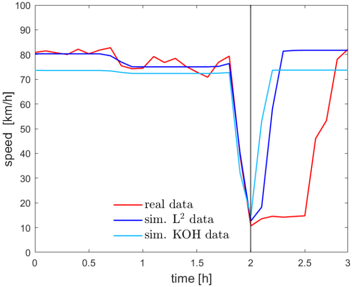
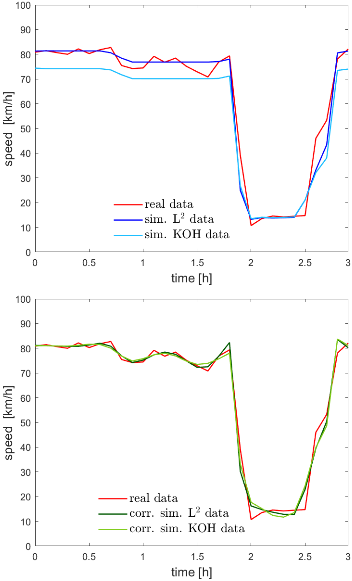


Traffic prediction.



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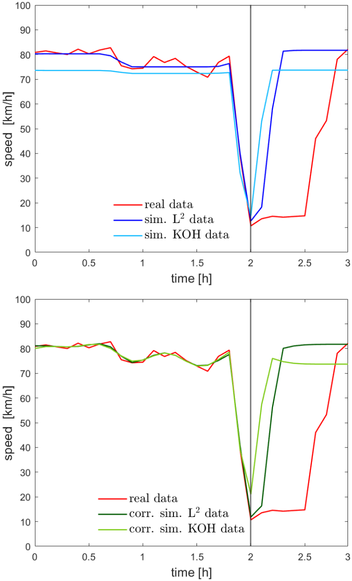

Traffic prediction.

GP prediction

Standard approach: **predictive mean** formula

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Extension: **integrate PDE** into GP modeling

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→ Variational posterior distribution with large set of hyperparameters

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On going work: consideration of PDE constraint in hyperparameter optimization

Travel time error

Consideration of $N = 875$ **vehicle trajectories** starting every 10 seconds.

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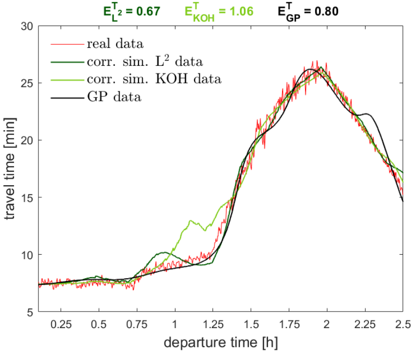
The **travel time error** is given by:

$$\mathrm{E}^{\mathrm{T}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\tau_i - \hat{\tau}_i)^2}$$

where $\hat{\tau}_i$ denotes the estimated (or predicted) travel time.



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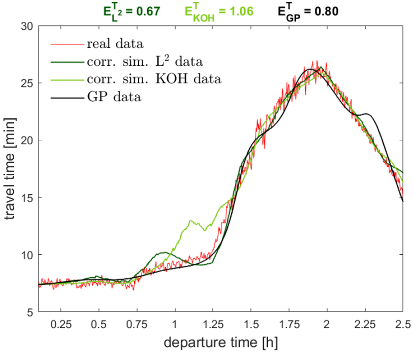
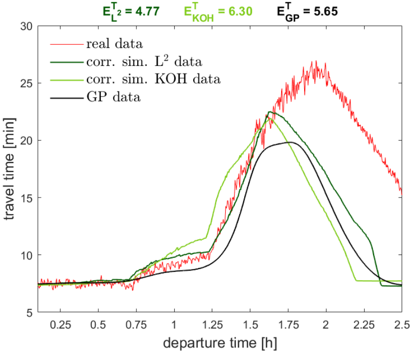


Traffic estimation.



Traffic prediction.

Conclusion

- Proposition of a statistical framework for **traffic state reconstruction**
- Introduction of a **discrepancy term** to compensate model limitations

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Outlook

- Consideration of real traffic data
- Improvement of boundary loop detector predictions

Thank you

Any questions?

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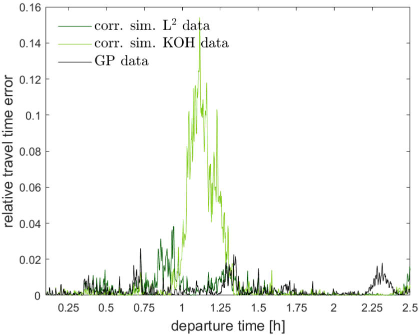
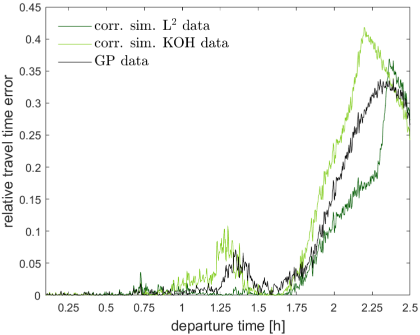
Travel time error

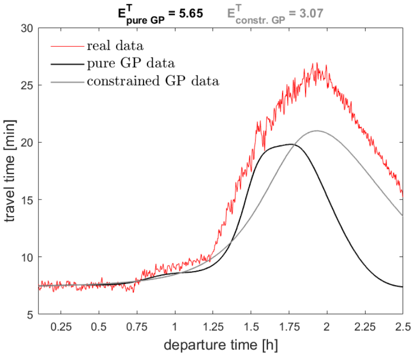


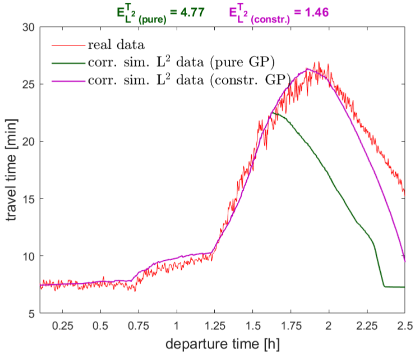
Traffic estimation.



Traffic prediction.

Two methods of GP prediction





Definition (Weak entropy solution)

A function $W \in \mathrm{L}^\infty\big((]0, T[\times]x_{in}, x_{out}[) \, ; \mathcal{W} \big)$ is a weak entropy solution to the IBVP if

- for any entropy-flux pair $(\mathcal{E}, \mathcal{Q})$ and any test function $\phi \in \mathrm{C}^\infty_c\big((]-\infty, T[\times]x_{in}, x_{out}[) \, ; \mathbb{R}_{\geq 0} \big)$, it holds

$$\int_0^T \int_{x_{in}}^{x_{out}} \{\mathcal{E}(u(W))\partial_t\phi + \mathcal{Q}(u(W))\partial_x\phi\} \, \mathrm{d}x\mathrm{d}t + \int_{x_{in}}^{x_{out}} \mathcal{E}(u(W_0(x)))\phi(0,x) \, \mathrm{d}x \geq 0;$$

- for any boundary entropy pair (α, β) and any $\gamma(t) \in \mathrm{L}^1\,(]0, T[; \mathbb{R}_{\geq 0})$ it holds

$$\operatorname*{ess\,lim}_{x \to x_{in}+} \int_0^T \beta(u(W(t,x)), u(W_{in}(t)))\gamma(t)\mathrm{d}t \leq 0,$$
$$\operatorname*{ess\,lim}_{x \to x_{out}-} \int_0^T \beta(u(W(t,x)), u(W_{out}(t)))\gamma(t)\mathrm{d}t \geq 0.$$

Definition (Boundary entropy pair)

An entropy pair $(\alpha(u_1, u_2), \beta(u_1, u_2))$, $u_1, u_2 \in \mathbb{R}^2$ is called a boundary entropy pair[7] if for every fixed $u_2 \in \mathbb{R}^2$

$$\alpha\,(u_2, u_2) = \beta\,(u_2, u_2) = \nabla_1 \alpha\,(u_2, u_2) = (0,0)^\top.$$

[7] Chen, G. and Frid, H., *Divergence Measure Fields and Hyperbolic Conservation Laws*, 1999

The **entropy boundary conditions** with respect to the left (resp. right) boundary state $W_B = (v_B, w_B)$ and for $j \in \{1, 2\}$ reads as follows:

$$\beta^j(W, W_B) := \mathcal{Q}^j(W) - \mathcal{Q}^j(W_B) - \nabla_u \mathcal{E}^j(W_B) \cdot (f(W) - f(W_B)) \leq (\geq) 0.$$

We consider the following families of **entropy-flux pairs**[8]:

$$\mathcal{E}^1(u(W)) = \begin{cases} 0 & \text{if } v \leq \bar{v}, \\ 1 - \frac{\mathcal{R}(v,w)}{\mathcal{R}(\bar{v},w)} & \text{if } v > \bar{v}, \end{cases}$$

$$\mathcal{Q}^1(u(W)) = \begin{cases} 0 & \text{if } v \leq \bar{v}, \\ \bar{v} - \frac{v\mathcal{R}(v,w)}{\mathcal{R}(\bar{v},w)} & \text{if } v > \bar{v} \end{cases}$$

for any $\bar{v} \in [0, w_{max}]$ and[9]

$$\mathcal{E}^2(u(W)) = \mathcal{R}(v, w)|\bar{w} - w|$$
$$\mathcal{Q}^2(u(W)) = \mathcal{R}(v, w)v|\bar{w} - w|$$

for any $\bar{w} \in [w_{min}, w_{max}]$.

[8] Andreianov, B.,Donadello, C. and Rosini, M. D., *A second-order model for vehicular traffics with local point constraints on the flow*, 2016

[9] Serre, D., *Systemes de Lois de Conservation*, 1996